Chapter 1
Basic Probability Review

1.1.

(a) For (n_1, n_2, n_3), let n_i be the number of demands at ith distribution center
$\Omega = \{(0,0,0),\ (1,0,0),\ (2,0,0),\ (0,1,0),\ \cdots,\ (2,2,2)\}$
(b) 2^{27}
(c) { (0,1,2), (0,2,1), (1,1,1), (1,0,2), (1,2,0), (2,0,1), (2,1,0)}
(d) $\frac{7}{27}$

1.3.
$\Pr\{N = 0\} = 0.82$
$\Pr\{N = 1\} = 0.15$
$\Pr\{N = 2\} = 0.03$

1.5.

(a) $k = 2$
(b) 0.6322
(c) 0.8646
(d)

$$F(t) = \begin{cases} 0 & \text{for } t < 0.5 \\ 1 - e^{-2t+1} & \text{for } t \geq 0.5 \end{cases}$$

(e)

$$\Pr\{T_1 + T_2 \leq t\} = 1 - e^{-2(t-1)} - 2(t-1)e^{-2(t-1)} \quad \text{for } t \geq 1\ .$$

Therefore, $\Pr\{T_1 + T_2 \leq 2\} = 1 - 3e^{-2} = 0.5940$.
(f) Let φ be the pdf for Y, then

$$\varphi(t) = \begin{cases} 0 & \text{for } t < 0.5 \\ 2t - 2 + e^{-2t+1} & \text{for } 0.5 \leq t < 1.5 \\ e^{-2t+1}(1 + e^2) & \text{for } t \geq 1.5\ . \end{cases}$$

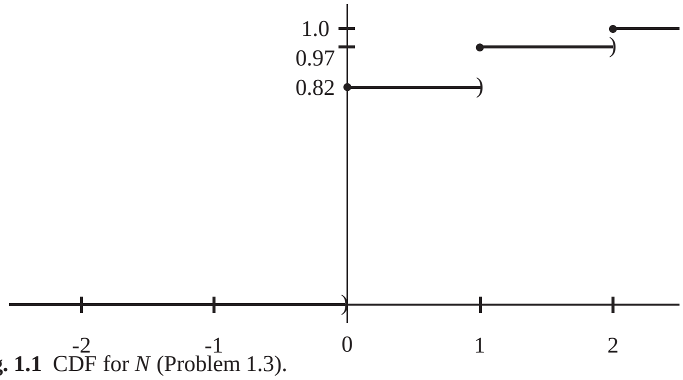

Fig. 1.1 CDF for N (Problem 1.3).

1.7.
Let P be the random variable denoting the profit
$E[P] = 510$

1.9.

Plan	Mean	Std. Dev.
A	27500	0
B	27600	489.9
C	28200	3340.7

1.11.

$$\begin{aligned} E[(X-u)^2] &= E[X^2 - 2\mu X + \mu^2] \\ &= E[X^2] - 2\mu E[X] + \mu^2 \qquad (\because\ E[X] = \mu) \\ &= E[X^2] - 2\mu^2 + \mu^2 \\ &= E[X^2] - \mu^2 \end{aligned}$$

1.13.

$$\begin{aligned} E[X] &= \int_0^b [1-F(x)]\,dx = \int_{x=0}^{\infty} [1-F(x)]\,dx \\ &= \int_{x=0}^{\infty}\int_{t=x}^{\infty} f(t)\,dt\,dx = \int_{t=0}^{\infty}\int_{x=0}^{t} f(t)\,dx\,dt \\ &= \int_{t=0}^{\infty} f(t)\left(\int_{x=0}^{t} dx\right) dt = \int_{t=0}^{\infty} t\,f(t)\,dt \end{aligned}$$

1.15.
The number of defective parts in a box follows the Binomial Distribution.
Let N denote the number of defective parts in a box.

(a) $\Pr\{N = 0\} = 0.8587$

(b) $\Pr\{N=2\} = 8.214 \times 10^{-3}$
(c) $\Pr\{N \geq 2\} = 8.506 \times 10^{-3}$
(d)

$$Pr\{N \geq 4\} = 0.03377$$

(e) $\Pr\{N \geq 20\} = 1 - \Pr\{N < 19.5\} \approx 1 - \Pr\{Z \leq \frac{19.5-12}{3.41}\} = 0.0139$

1.17.
Let θ denote the angle
$\Pr\{9.9 \leq \theta \leq 10.1\} = \frac{0.2}{0.8} = 0.25$

1.19.

(a) $\Pr\{sick \mid male, treated\} = \frac{200}{500} = 0.4$
(b) If only consider the population of males, the treatment should be recommended.
(c) If only consider the population of females, the treatment should be recommended.
(d) If consider the entire population, the treatment should not be recommended.

1.21.

(a) $k = 6$
(b) Let f_1 be the marginal pdf for S

$$f_1(s) = 2s \qquad \text{for} \quad 0 \leq s \leq 1$$

Let f_2 be the marginal pdf for T

$$f_2(t) = 3t^2 \qquad \text{for} \quad 0 \leq t \leq 1$$

$\Pr\{S \leq 0.5\} = 0.25$

$E[S] = \frac{2}{3}$
(c)

$$f_{1|t} = 2s$$

$\Pr\{S \leq 0.5 \mid T = 0.1\} = 0.25$

$E[S \mid T = 0.1] = \frac{2}{3}$
(d) S and T are independent
$\because \quad f(s,t) = f_1(s) \cdot f_2(t)$ for all s and t.

1.23.

(a) $\Pr\{T \geq \frac{3}{12}\} = 0.875$

(b) $\Pr\{T \geq 1.25 \mid T \geq 1\} = 0.75$
(c) Now $T \sim Expo(1)$
$\Pr\{T \geq \frac{3}{12}\} = 0.7788$
$\Pr\{T \geq 1.25 \mid T \geq 1\} == 0.7788$

(d) For the exponential distribution, it is not important to know how old of the machine is because of its memoryless property; however, for any other continuous distribution, it is important to know the machine's age, as is illustrated by the fact that the answers to part (a) and (b) differ.

1.25.

(a) The expected number is $1000 \times (0.0668 + 0.3085) = 375.3$.
(b) The expected number is $1000 \times 0.2119 = 211.9$.
(c) $\Pr\{B < 124 | A = 5.94\} \approx 0.5529$.
(d) $\Pr\{B < 124 | A = 6.08\} \approx 0.0415$.

1.27.

(a) The joint pmf is

	$S=0$	$S=1$	$S=2$
$X_1 = 0$	0.8	0.12	0
$X_2 = 1$	0	0.32	0.48

(b) $\Pr\{S=0\} = 0.08$, $\Pr\{S=1\} = 0.44$, and $\Pr\{S=2\} = 0.48$.
(c)

$$\rho = 0.632\,.$$

(d) The conditional pmf for S given $X_1 = 0$ is

$$\Pr\{S=0|X_1=0\} = 0.4, \quad \Pr\{S=1|X_1=0\} = 0.6, \quad \Pr\{S=2|X_1=0\} = 0\,,$$

and the conditional pmf for S given $X_1 = 1$ is

$$\Pr\{S=0|X_1=1\} = 0, \quad \Pr\{S=1|X_1=1\} = 0.4, \quad \Pr\{S=2|X_1=1\} = 0.6\,.$$

(e) $E[S|X_1=0] = 0.6$ and $E[S|X_1=1] = 1.6$; therefore,

$$E[S] = 0.2 \times 0.6 + 0.8 \times 1.6 = 1.4\,.$$

(f) $V[S|X_1=0] = 0.24$ and $V[S|X_1=1] = 0.24$; therefore, $E[V[S|X]] = 0.24$. Also,

$$V[E[S|X]] = (0.2 \times 0.6^2 + 0.8 \times 1.6^2) - 1.4^2 = 0.16$$

and thus we have $V[S] = 0.24 + 0.16 = 0.4$.