Chapter 2
Basics of Monte Carlo Simulation

The difficult part of doing simulations by hand is establishing the column headings. Of course such headings are not unique, but we give below suggestions for the headings and some entries for the problems that do not have tables already given by example in the text. For random numbers, students may use either the random numbers on their calculators or from Excel.

2.1.

Random laws
    coin toss:
        H $\rightarrow$ gets in; T $\rightarrow$ does not get in
    die toss:
        1 $\rightarrow$ \$0; 2,3,4 $\rightarrow$ \$60; 5,6 $\rightarrow$ \$100

Time	Coin toss	Gets in?	Die toss	Sales price	Total	Avg.
1	H	Y	2	60	60	60
2	H	Y	6	100	160	80
3	T	N	–	0	160	53.33
4	H	Y	1	0	160	40
5	T	N	–	0	160	32
6	H	Y	5	100	260	43.33
$\vdots$						$\vdots$

Theoretical value $= 0.5 \times (0.5 \times 60 + 0.333 \times 100) = \$31.67$

2.3.
Since the problem gives the modulus, only the multiplier need be given to define the generator. Anyone of the following will produce a full period: 3, 11, 12, 13, 17, 21, 22, 24.

2.5.

Schrage suggested $m = 2^{31} - 1$ and $a = 16,807$. The first three seeds and random numbers are $S_1 = 1,697,507$; $RN_1 = 0.0008$; $S_2 = 612,712,738$; $RN_2 = 0.2853$; $S_3 = 678,900,201$; $RN_3 = 0.3161$.

2.7.

Three iterations of L'Ecuyer's generator produces the following

k	S_k	T_k	U_k	RN
-2	376810349	3650079346		
-1	1545165407	1898051052		
0	3443838735	2606578666		
1	4169848126	774095269	3395752857	0.7906
2	1078292449	2756964901	2616294635	0.6092
3	2317335712	4196546513	2415756286	0.5625

2.9.

There is more than one way to simulate most problems and this is no exception. For illustrative purposes, we suggest the all five items be placed in the box and then sampled. Two tables will have to be built: one to build the boxes and one to sample the boxes.

Conversion from random numbers to events:

$[0.0, 0.2) \rightarrow$ bad item; $[0.2, 1.0] \rightarrow$ good item

$\lfloor 1 + RN \times 5 \rfloor =$ number of item selected

(Note that $\lfloor \cdot \rfloor$ denotes truncation.) If the item of the second item selected equals the first item selected, then another random number will be selected.

The following is the table giving the simulation of the packaging of the boxes. The random number indicates whether or not an item that was placed in the box is good or bad.

Box number	Random numbers	Status of items in box (listed in order)
1	0.9019, 0.3689, 0.2013, 0.8849, 0.5020	g, g, b, g , g
2	0.4779, 0.5881, 0.9000, 0.7879, 0.7121	g, g, g, g , g
3	0.1472, 0.9092, 0.3621, 0.7132, 0.6690	b, g, g, g , g
4	0.3953, 0.1110, 0.8129, 0.6077, 0.1828	g, b, g, g , b
5	0.5929, 0.3456, 0.2016, 0.9739, 0.2822	g, g, g, g , g
⋮		⋮

The following is the simulation of the inspection process and the shipping of the boxes that passed inspection. The random number indicates which item is selected. If the second random number generated indicates the same item as the first, then another random number is generated. The box number and the indication of which item is bad come from the first table.

Box number	Random numbers	Items selected	Box shipped	Cum. defects	Cum. boxes
1	0.4703, 0.8872	#3, #5	no	—	—
2	0.5035, 0.2335	#3, #2	yes	0	1
3	0.6576, 0.5989, 0.2047	#4, #2	yes	1	2
4	0.4328, 0.2419	#3, #2	no	1	2
5	0.1215, 0.6602	#1, #4	yes	1	3
⋮					⋮

2.11.

(a) 5.878, 1.204, 2.248, 10.637   (It would also be acceptable to have the values 1.045, 5.388, 3.376, 0.290 since either mean$\times\ln(R)$ or mean$\times\ln(1-R)$ is technically correct.)

(b) 4.666, 2.475, 3.177, and 5.915. (However, the transformation could have used $\ln(1-R)$ also, which would yield 2.339, 4.506, 3.738, and 1.401.)

(c) The random numbers using the Box Muller procedure depends on the pairing. When $R_1 = 0.23$ and $R_2 = 0.74$, we obtain standard normal variates of -0.185 and -2.934. When $R_1 = 0.57$ and $R_2 = 0.07$, we obtain standard normal variates of 1.017 and 0.479. Thus, the normal random variates with a mean of 4 and a standard deviation of 2 are 3.630, -1.868, 6.034, and 4.958.

(d) The data must be ordered and an empirical distribution drawn (see the final section in this chapter). Each data point is within a cell containing probability equal to 0.05 (i.e., $\frac{1}{\text{no. data points}}$). The empirical distribution is a piecewise linear function with linear interpolation occuring between the following points: (5.65,0.05), (6.85,0.10), (7.10,0.15), etc. The random variate are 8.0, 12.99, 11.56, 6.13.

2.15.

Note that the time units for both parts were chosen to be hours.

(a) $\alpha = 1.642$ and $\beta = 89.427$

(b) $\mu_N = 0.85536$ and $\sigma_N = 0.34909$

(c) $\rho \approx -0.81$.