

## Chapter 5

### Markov Chains

5.1.

(a)  $E = \{0, 1, 2, 3, 4\}$

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.48 & 0 & 0.52 & 0 & 0 \\ 0 & 0.48 & 0 & 0.52 & 0 \\ 0 & 0 & 0.48 & 0 & 0.52 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

(b) 0.27

(c) 0.27

(d) 0.5

(e) 2.08

(f) 0.54

(g) 3.993

5.3. (a)  $E = \{h, g, s, i, d, \}$

$$\mathbf{P} = \begin{matrix} & \begin{matrix} h & g & s & i & d \end{matrix} \\ \begin{matrix} h \\ g \\ s \\ i \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.55 & 0.3 & 0.15 & 0 & 0 \\ 0.1 & 0.2 & 0.55 & 0.1 & 0.05 \\ 0 & 0.05 & 0.3 & 0.55 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

(b) 0.63

(c) 2.676

(d) 2.395

(e) 54.93

5.5. The average cost per item shipped is \$715.52.

5.7. (a)  $E = \{b, s, c, o\}$

$\mu = (0.344, 0.205, 0.235, 0.216)$  (based on June totals)

$$\mathbf{P} = \begin{matrix} & \begin{matrix} b \\ s \\ c \\ o \end{matrix} \\ \begin{matrix} b \\ s \\ c \\ o \end{matrix} & \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0.15 & 0.05 & 0.6 & 0.2 \\ 0.3 & 0.1 & 0.2 & 0.4 \end{bmatrix} \end{matrix}$$

(b) 15.96

(c) 18.81

5.9. (a)  $E = \{0, 1, 2, 3, 4, 5\}$ 

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0.15 & 0.1 & 0.4 & 0.2 & 0.1 & 0.05 \\ 0.15 & 0.1 & 0.4 & 0.2 & 0.1 & 0.05 \\ 0.15 & 0.1 & 0.4 & 0.2 & 0.1 & 0.05 \\ 0.65 & 0.2 & 0.1 & 0.05 & 0 & 0 \\ 0.25 & 0.4 & 0.2 & 0.1 & 0.05 & 0 \\ 0.15 & 0.1 & 0.4 & 0.2 & 0.1 & 0.05 \end{bmatrix} \end{matrix}$$

(b) 0.05

(c) 0.035

(d) 37.2

(e) Avg. long-run profit, old policy = \$1801

Avg. long-run profit, proposed policy = \$1781; therefore, not worthwhile

5.11. (a) 0.1

(b) 0.39

(c) 0.39

(d) 0.417

(e) 0.04

(f) 21

(g) 0.203

(h) 0.443

(i) 21.52

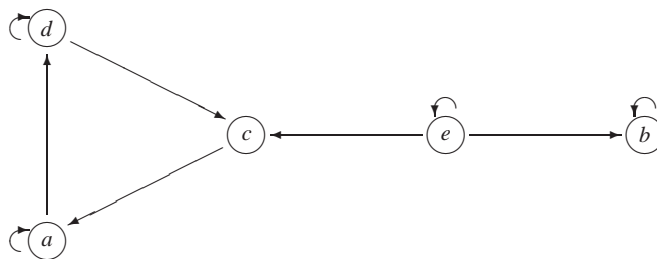


Fig. 5.1 State diagram for the Markov chain of Problem 5.13

5.13. (a)

(b)  $\{e\}$ (c)  $\{a, c, d\}, \{b\}$ 

(d)

$$\mathbf{F} = \begin{matrix} a \\ d \\ c \\ b \\ e \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.667 & 0.667 & 0.667 & 0.333 & 0.4 \end{bmatrix}$$

(e)

$$\mathbf{R} = \begin{matrix} a \\ d \\ c \\ b \\ e \end{matrix} \begin{bmatrix} \infty & \infty & \infty & 0 & 0 \\ \infty & \infty & \infty & 0 & 0 \\ \infty & \infty & \infty & 0 & 0 \\ 0 & 0 & 0 & \infty & 0 \\ \infty & \infty & \infty & \infty & 1.667 \end{bmatrix}$$

(f)

$$\lim_{n \rightarrow \infty} \mathbf{P}^n = \begin{matrix} a \\ d \\ c \\ b \\ e \end{matrix} \begin{bmatrix} 0.323 & 0.452 & 0.226 & 0 & 0 \\ 0.323 & 0.452 & 0.226 & 0 & 0 \\ 0.323 & 0.452 & 0.226 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.215 & 0.301 & 0.151 & 0.333 & 0 \end{bmatrix}$$

**5.15.** (a) 0.417

(b) 0.417

(c) 0

(d) 0.208