Chapter 7
Queueing Processes

7.1. (a) $15 \times 0.5 = 7.5$

(b) $\frac{1}{15}$ hr, or 4 min

(c) $\frac{18}{60} \times 15 = \frac{9}{2}$

(d) 0.1125

(e) 0.0111

(f) 11 min

7.3. (a) $\frac{3}{4}$

(b) $\frac{9}{4}$

(c) $\frac{9}{16}$

(d) $\frac{1}{5}$

(e) Cost/hr = 14.4/hr

Cost'/hr = 14.8/hr

Therefore, do not use the additional expenditure.

7.5. (This problem uses the closed form of the geometric progression as shown in the footnote by Eqs. (5.5) and (5.7).)

(a) For birth-death equations, let $\lambda_n = \lambda$ for all n, and let $\mu_1 = \mu, \mu_2 = 2\mu, \mu_n = 3\mu$ for $n \geq 3$. Thus,

$$
\begin{aligned}
p_1 &= p_0 \frac{\lambda}{\mu} \\
p_2 &= p_0 \frac{\lambda \cdot \lambda}{\mu \cdot 2\mu} \\
p_3 &= p_0 \frac{\lambda \cdot \lambda \cdot \lambda}{\mu \cdot 2\mu \cdot 3\mu} \\
p_4 &= p_0 \frac{\lambda \cdot \lambda \cdot \lambda \cdot \lambda}{\mu \cdot 2\mu \cdot 3\mu \cdot 3\mu} \\
\vdots &\quad \vdots
\end{aligned}
$$

We can show by induction that $p_n = p_0 \times \lambda^n/(2\mu^n \cdot 3^{n-2})$ for $n \geq 2$, and the equations $p_1 = 3\rho p_0$ and $p_n = 4.5\rho^n p_0$ for $n \geq 2$. Then the norming equation is used for p_0:

$$\begin{aligned}
1 &= \sum_{n=0}^{\infty} p_n \\
&= p_0 + 3\rho p_0 + \sum_{n=2}^{\infty} 4.5\rho^n p_0 \\
&= p_0 + 3\rho p_0 + 4.5\rho^2 p_0 \sum_{n=0}^{\infty} \rho^{n-2} \\
&= p_0 \left(1 + 3\rho + 4.5\rho^2 \times \frac{1}{1-\rho}\right) \\
&= p_0 \frac{1 + 2\rho + 1.5\rho^2}{1-\rho}
\end{aligned}$$

The result follows after solving for p_0.

(b)

$$\begin{aligned}
L_q &= \sum_{n=1}^{\infty} n p_{n+3} \\
&= 4.5 p_0 \sum_{n=1}^{\infty} n\rho^{n+3} \\
&= 4.5 p_0 \sum_{n=1}^{\infty} n\rho^{n+3} \\
&= 4.5 p_0 \rho^4 \sum_{n=1}^{\infty} n\rho^{n-1} \\
&= 4.5 p_0 \rho^4 \frac{1}{(1-\rho)^2}
\end{aligned}$$

The result follows after substituting for p_0 and simplifying.

7.7. (a) $\lambda = 6$ /day $\quad \mu = 4.8$ /day and $\rho = 1.25$

For M/M/1/3 system,

$$p_3 = (\frac{5}{4})^3 \frac{1 - \frac{5}{4}}{1 - (\frac{5}{4})^4} = 0.339$$

$$\begin{aligned}
L_{k=3} &= \frac{5}{4} \times \frac{1 + 3\,(\frac{5}{4})^4 - 4 \times (\frac{5}{4})^3}{(1-\frac{5}{4})(1-(\frac{5}{4})^4)} \\
&= 1.775
\end{aligned}$$

$\text{Cost}_{k=3}/\text{day} = 30 \times 1.775 + 105 \times 6 \times 0.339 = 266.82\,/\text{day}$

For M/M/1/4 system,

$$p_4 = (\frac{5}{4})^4 \frac{1-\frac{5}{4}}{1-(\frac{5}{4})^5} = 0.297$$

$$L_{k=4} = \frac{5}{4} \times \frac{1+4\,(\frac{5}{4})^5 - 5 \times (\frac{5}{4})^4}{(1-\frac{5}{4})(1-(\frac{5}{4})^5)} = 2.437$$

$\text{Cost}_{k=4}/\text{day} = 30 \times 2.437 + 105 \times 6 \times 0.297 = 260.22\,/\text{day}$
Therefore, the proposed policy change is better.
(b) For M/M/1/5 system,

$$p_5 = (\frac{5}{4})^5 \frac{1-\frac{5}{4}}{1-(\frac{5}{4})^6} = 0.271$$

$$L_{k=5} = \frac{5}{4} \times \frac{1+5\,(\frac{5}{4})^6 - 6 \times (\frac{5}{4})^5}{(1-\frac{5}{4})(1-(\frac{5}{4})^6)} = 3.132$$

$\text{Cost}_{k=5}/\text{day} = 30 \times 3.132 + 105 \times 6 \times 0.271 = 264.69\,/\text{day}$
The optimum cut-off policy is to allow 4 jobs in center.
(c) $6 \times 105 = 630\,/\text{day}$
400 + 260.22 = 660.22 /day ($>$ 630 /day)
$\therefore$ Use only the outside contractor.
(d)

7.9. (a) For the M/M/1 system, $\lambda = 100\,/\text{day}$ $\mu = 144\,/\text{day}$ and $\rho = \frac{25}{36}$
$L_{c=1} = 2.273$
For the M/M/3 system, $\lambda' = 100\,/\text{day}$ $\mu' = \frac{60\times 24}{27}\,/\text{day}$ and $r = \frac{15}{8}$
$p_0 = 0.1322$
$L_q = 0.1322 \times \frac{3\times(\frac{15}{8})^4}{6\times(3-\frac{15}{8})^2} = 0.6455;\quad L_{c=3} = L_q + r = 2.5205$

$\therefore$ The robot reduces the inventory most.
(b) $0.5 \times 24 = 12\,/\text{day}$
For the M/M/2 system, $\lambda'' = 100\,/\text{day}$ $\mu'' = \frac{160}{3}\,/\text{day}$ and $\rho'' = \frac{15}{16}$
$L_{c=2} = \frac{2\rho''}{1-\rho''^2} = 15.48$

$\text{Cost}_{c=1}/\text{yr.} = 12 \times 2.273 \times 365 + 100{,}000 = 109{,}955.74/\text{yr.}$
$\text{Cost}_{c=2}/\text{yr.} = 12 \times 15.48 \times 365 + 40{,}000 \times 2 = 147{,}821.45/\text{yr.}$

$\text{Cost}_{c=3}/\text{yr.} = 12 \times 2.5205 \times 365 + 40,000 \times 3 = 131,039.79.79/\text{yr.}$
$\therefore$ Still use the robot.

7.11. The M/M/∞ system was not discussed in the chapter, so the first step will have to be to derive the equations for this system.

(a) $\frac{(\frac{\lambda}{\mu})^n e^{-\frac{\lambda}{\mu}}}{n!}$.
(b) $L = \frac{\lambda}{\mu}$ $\because$ the distribution given in part (a) is Poisson
(c) $\frac{\lambda}{\mu}$ (Again this is not derived since the distribution in part (a) should be recognized as being Poisson.)
(d) $L_q = 0$
(e) $\frac{1}{\mu}$

7.13. (a) $p_5 = 0.138$
(b) $E[\# \, working] = 1.927$
(c) $\lambda_e = 0.48175/\text{day}$
$W = 6.379$

7.15. The data of interarrival times yield: mean = 5.72 min. and st.dev = 4.70 min.

(a) $W_q = 7.5$
$L = 1.92$
(b) $W_{q,wrong} = 5.52 \Longrightarrow$ 26% error.
$L_{wrong} = 1.57 \Longrightarrow$ 18% error.
(c) Current cost: = $19.67/hr
Bar reader cost: = $7.16/hr. Assume that the cost savings occurs 86 hours per month yielding a cost reduction of approximately $1,076 yielding a net savings of $876 per month.
New hire cost: = $6.97/hr. Again, assume that the cost savings occurs 86 hours per month yielding a cost reduction of approximately $1,092 yielding a net savings of $742 per month. Thus, the bar reader is better economically; however, hiring another individual may be better when considering other factors that were not reduced to numerical values.