Chapter 9
Event-Driven Simulation and Output Analyses

The hand simulations need at least two charts: the future events list and the main simulation chart. My guess is that the future events list should not prove difficult, but the main chart may be more difficult, principally due to the figuring the headings. Of course such headings are not unique, but we give below the headings and some entries for the problems that do not have tables already given by example in the text.

Problems designed for Excel are not included here.

9.1. The first problem is simply a repeat of the example, except with different random numbers. Therefore, it will not be repeated here.

9.3. Random laws → same as in textbook except
die toss: 1 → 10;   2,3 → 11;   4,5,6 → 12.

Clock time	Type of event	die toss	Time next arrival	Two coin toss	Good or bad item	Number in system	Three coin toss	Time service complete	Sum of time × number
0	a	2	11	TT	bad	0	–	–	0
11	a	4	23	TH	good	1	THT	27	0
23	a	2	33	HH	good	2	–	–	12
27	s	–	–	–	–	1	HHH	42	20
33	a	1	43	HT	good	2	–	–	26
42	s	–	–	–	–	1	TTT	56	44
43	a	2	54	HH	good	2	–	–	45
⋮									⋮

9.5. Random laws

coin toss:
HH → 345 seconds and $10
HT,TH → 350 seconds and $10.50
TT → 355 seconds and $11
coin toss: (interarrival times)
HH → 150 seconds; HT,TH → 180 seconds; TT → 210 seconds

Time	Type event	Coin toss	Time next arrival	Number in system	Coin toss	Service time	Time depart	Sum of profit	Time × number	Sum of time × number
0	a	HH	150	1	HT	350	350	10.50	–	0
150	a	TH	330	2	HH	345	495	20.50	150	150
330	a	TT	540	3	–	–	–	20.50	360	510
350	s	–	–	2	TT	355	705	31.50	60	570
495	s	–	–	1	–	–	–	31.50	290	860
540	a	TT	750	2	HH	345	885	41.50	45	905
⋮										⋮

9.7. Using the probabilities from Problem 2.1, we have the following random laws:

Random laws:

$R \in [0.0, 0.5833) \rightarrow P = 0$
$R \in [0.5833, 0.8333) \rightarrow P = 60$
$R \in [0.8333, 1.0] \rightarrow P = 100$

The following are the value I get using the random number generator on my calculator: 0, 100, 60, 100, 0, 0, 60, 100, 0, 0, 100, 0, 0, 60, 0, 100, 60, 0, 0, 0, 0, 0, 60, 0, 60. The sum of the values and the sum of squares are

$$\sum_i x_i = 960 \quad \sum_i x_i^2 = 71,600\,.$$

Thus, the sample mean and standard deviation are

$$\overline{x} = 38.4 \quad s = \sqrt{1447.333} = 38.0\,.$$

Therefore, the confidence interval is

$$38.4 \pm 2.064 \frac{38.0}{5} \Rightarrow (22.71, 54.09)\,.$$

The theoretical values are

$$\mu = 0.25 \times 60 + 0.167 \times 100 = 31.7$$

which is within the confidence interval.

9.11. (a)

$$\begin{aligned} E[T_0] &= E[T_0|X_0 = a]P\{X_0 = a\} + E[T_0|X_0 = b]P\{X_0 = b\} \\ &= 0.5\theta_a + 0.5\theta_b \end{aligned}$$

(b) First define η_i to be second moment in each state; that is,

$$\sigma_i^2 = \eta_i - \theta_i^2\,.$$

$$
\begin{aligned}
E[T_0^2] &= E[T_0^2|X_0=a]P\{X_0=a\}+E[T_0^2|X_0=b]P\{X_0=b\} \\
&= 0.5(\eta_a^2+\eta_b^2) \text{ therefore} \\
var(T_0) &= 0.5(\sigma_a^2+\theta_a^2+\sigma_b^2+\theta_b^2)-E[T_0]^2 \\
&= 0.5(\sigma_a^2+\sigma_b^2)+0.5(\theta_a^2+\theta_b^2)-0.25(\theta_a^2+2\theta_a\theta_b+\theta_b^2) \\
&= 0.5(\sigma_a^2+\sigma_b^2)+0.25(\theta_a^2-2\theta_a\theta_b+\theta_b^2)
\end{aligned}
$$

(c)

$$
\begin{aligned}
E[T_0T_1] &= \sum_{i,j}\theta_i\theta_j P\{X_0=i,X_1=j\} \\
&= \theta_a\theta_a\times 0.5p+\theta_a\theta_b\times 0.5q+\theta_b\theta_a\times 0.5q+\theta_b\theta_b\times 0.5p \\
&= E[T_0T_1|X_0=a]P\{X_0=a\}+E[T_0T_1|X_0=b]P\{X_0=b\} \\
E[T_0T_1]-E[T_0]^2 &= 0.5((p-0.5)\theta_a^2+2(q-0.5)\theta_a\theta_b+(p-0.5)\theta_b^2) \\
&= 0.5(p-0.5)(\theta_a^2-2\theta_a\theta_b+\theta_b^2) \\
&= 0.25(2p-1)(\theta_a=\theta_b)^2
\end{aligned}
$$

Dividing the above equation by the answer from part (b) yields the desired result.

9.13. The simulation of these correlated random variables is contained in the Excel file associated with this chapter. Note that this is also an implementation of Property 2.6.